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LESSON 1

REAL NUMBERS

Real Numbers

- Rational Numbers
  - Integers
    - Natural Numbers
    - Zero and Negative integers
  - Fractions of the type $\frac{p}{q}$, where p, q are positive integers and $q \neq 0$ and HCF of p and q is 1
- Irrational Numbers

- **IMPORTANT TERMS**

Natural numbers: Counting numbers are known as natural numbers.
Thus, 1, 2, 3, 4, 5, 6, 7, ..., etc., are all natural numbers.

Whole numbers: All natural numbers together with 0 form the collection of all whole numbers.

Integers: All natural numbers, 0 and negatives of natural numbers form the collection of all integers.

Rational numbers: A number 'r' is said to be a rational number, if it can be expressed in the form $\frac{p}{q}$, where p and q are integers and $q \neq 0$.

1. Euclid's division algorithm or Euclid's division Lemma

Given positive integers a and b, there exists unique integers q and r satisfying $a = bq + r$, $0 \leq r < b$.

Let a and b be positive integers, If $a = bq + r$, $0 \leq r < b$, then

$$\text{HCF}(a,b) = \text{HCF}(b,r).$$

(i) Finding HCF of two numbers a, b. $(a > b)$ using Euclid's Division Algorithm

Step 1: Write $a = bq + r$ satisfying the relation $0 \leq r < b$.

Step 2: Write $b = pr + s$ satisfying the relation $0 \leq s < r$.

Step 3: Repeat the process till the remainder becomes zero. Then, the divisor of the last step would be the HCF.

For example:

To find HCF of 575 and 15 using Euclid's algorithm

$$575 = 15 \times 38 + 5$$

Now, consider 15 and 5 and applying Euclid's algorithm again.

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$15 = 5 \times 3 + 0$

Here, the remainder is zero.

$\therefore$ HCF of 15 and 5 is 5.

$\therefore$ HCF of 575 and 15 is also 5.

2. Fundamental theorem of Arithmetic

Every composite number can be expressed as product of primes and this expression is unique, apart from the order in which prime factors occur.

(i) Finding the HCF Using Prime Factorisation Method

Step 1: Find the prime factorization of the given numbers.

Step 2: Take out the common prime factors and multiply them that will give the HCF of those numbers

(ii) Finding the LCM Using Prime factorization Method

Step 1: Write the prime factors of each number.

Step 2: Write all different prime factors and common factors [use the factor of greatest power].

Step 3: Compute the product of all these factors which is the required LCM.

- HCF is the lowest power of common prime and LCM is the highest power of primes.

For example:

To find HCF and LCM of 45, 75 and 125 by prime factorization method.

$45 = 3 \times 3 \times 5 = 3^2 \times 5$

$75 = 3 \times 5 \times 5 = 3 \times 5^2$

$125 = 5 \times 5 \times 5 = 5^3$

$\text{HCF} = 5^1 = 5$

$\text{LCM} = 3^2 \times 5^3 = 9 \times 125 = 1125$

3. Irrational Numbers

(a) A number is said to be irrational if it cannot be written in the form $\frac{p}{q}$, where p and q are integers and $q \neq 0$.

(b) Any non perfect square root is an irrational number

4. Terminating and non-terminating numbers

Rational number	Form of prime factorisation of the denominator	Decimal expansion of rational number
$x = \frac{p}{q}$, where p and q are co-prime and $q \neq 0$	$q = 2^m 5^n$ where n and m are non-negative integers	terminating
	$q \neq 2^m 5^n$ where n and m are non-negative integers	non-terminating

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For example:
Without actual division, state whether the following rational numbers will have a terminating decimal expansion or non-terminating repeating decimal expansion.

(i) $\frac{17}{512}$

$\frac{17}{512} = \frac{17}{2^9}$

Since the denominator is in the form of $2^n 5^m$ and 9 > 0,

$\therefore$ $\frac{17}{512}$ will have terminating decimal.

(ii) $\frac{64}{455}$

$\frac{64}{455} = \frac{2^6}{5 \times 7 \times 13}$

Since the denominator is not in the form of $2^n 5^m$,

$\therefore$ $\frac{64}{455}$ will have non-terminating repeating decimal.

- Any positive integer can be expressed in the form of $3q$, $3q + 1$ or $3q + 2$ where q is some integer.

For example:
Show that the square of any positive integer is either of the form $3m$ or $3m + l$ for some integer m.

Type of n	n^2	Type of n^2
$3p$	$9p^2$	$3m$
$3p + 1$	$(3p+1)^2 = 9p^2 + 6p + 1 = 3(3p^2 + 2p) + 1$	$3m + 1$
$3p + 2$	$(3p+2)^2 = 9p^2 + 12p + 4$ $= 9p^2 + 12p + 3 + 1$ $= 3(3p^2 + 4p + 1) + 1$	$3m + 1$

- HCF × LCM = Product of two numbers

For example:
HCF of (16, 100) = 4, write LCM of (16, 100)

LCM = Product of two numbers

LCM = $\frac{16 \times 100}{4}$

LCM = 400.